6   Conclusion

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In this book, we have developed three overarching and interrelated ideas. The first of these ideas is that propositional conceptual contents are articulated by open reason relations, that is, by reason relations that need be neither monotonic nor transitive. These reason relations are relations of implication and incompatibility, and they constrain the norms that govern practices of giving and asking for reasons. In virtue of standing to one another in reason relations of implication and incompatibility, things are bearers of conceptual contents—specifically *propositional* contents. Those contents can be thought of as the functional roles the sentences play in constellations of implications and incompatibilities. Declarative sentences express what can both serve as and stand in need of reasons (for and against, positive and negative). This is a version of semantic inferentialism. Our semantic inferentialism includes a functionalism about conceptual contents: propositional conceptual contents are individuated by the role they play in reason relations. We first formulated semantic inferentialism in a pragmatic metavocabulary in Chapter One.

  The second idea is that we can make explicit these reason relations in terms of different metavocabularies. In particular, we can give accounts of reason relations in a pragmatic-normative metavocabulary and also in a semantic-representationalist metavocabulary. We call these kinds of metavocabulary "extrinsic" because they appeal to conceptual resources that are not provided by reason relations themselves. There are also two kinds of intrinsic metavocabularies, which use only conceptual resources provided by reason relations themselves. In particular, we can make reason relations explicit within the object language by using logical vocabulary, and we can also make them explicit in the metavocabulary of implication-space semantics. The idea is that these four metavocabularies afford us four perspectives on a common topic, namely reason relations. We thus undertook to understand reason relations as what shows up from the

perspective of all four metavocabularies. We called the roles played in reason relations that show up from all four perspectives "rational forms."

Our third overarching idea is that logic—appropriately understood—can serve as a guide to how the different perspectives on reason relations hang together. While it is a mistake to think that all reason relations are logical relations, logic is nevertheless intimately connected to reason relations. In particular, it is the essential and demarcating function of logical vocabulary to make explicit reason relations, and to do so within the language that is governed by these reason relations. This is the central thesis of logical expressivism, which we introduced in Chapter Two. More precisely, logical expressivism says that logical vocabulary is, firstly, vocabulary such that, for any language to which logical vocabulary is added, the reason relations among sentences in which logical vocabulary occurs can be computed from the reason relations among sentences in which logical vocabulary does not occur. And, secondly, logical vocabulary is necessary and sufficient to make explicit any reason relations among sentences of any language into which it is introduced. Thus, logic is universal in the sense that it can be explained independently of any particular reason relations of some non-logical base vocabulary. This allows us to investigate logical vocabulary from different perspectives without making assumptions about particular reason relations. Because of this expressive universality, logical vocabulary can serve as a focal point of our three other metavocabularies, thus making it easier for us to appreciate their connections.

In this final chapter, we look back at the results that we have reached by developing these three connected ideas. Before we do this, however, we want to repeat some general clarifications and caveats. First, our use of the term "reason" (we address doxastic, not practical reasons) differs from some current uses of the term. In particular, we think of implication as corresponding to the relation of being a reason for something, and we think of incompatibility as corresponding to the relation of being a reason against something. We do not assume, however, that if one accepts something that is a reason against something else, then one ought to reject the second thing. Nor do we assume that if one accepts a reason for something, then one may or ought to accept it. Rather, what we mean by "reason for" is that if A is a reason for B, then commitment to accept A precludes entitlement to reject B. And if A is a reason against B, then commitment to accept A precludes entitlement to accept B. Second, throughout the book we have restricted our attention to the propositional level and we have not said anything about subsentential expressions (apart from occasional asides).[1] Hence, there are many open questions about how our results generalize to subsentential expressions and, especially, quantifiers. Third, we want to stress that we think that the main force and importance of our technical results lies in the philosophically illuminating way in which they

connect different frameworks and perspectives, such as truth-maker theory and Restall's bilateralism or the generalization of Girard's phase-space semantics that is implication-space semantics. We do not understand our results as primarily contributions to mathematical propositional logic but rather as contributions to the philosophical understanding of propositional logic and how it is related to larger philosophical topics. Fourth, we have sometimes left open important issues and questions that have loomed large in philosophical logic, such as the issue of whether we should accept the structural rule of Contraction. We have, for instance, presented contractive and non-contractive versions of our sequent calculus and of implication-space semantics. Similarly, we have given short shrift to some influential problems in the theory of representation, such as the (alleged) under-determination of representation relations by functional roles. And we have not addressed many of the issues that arise in the current debate on the normativity of logic and which might be reproduced for the normativity of reason relations more generally, such as the question how (if at all) reason relations can sometimes make particular acceptances or rejections obligatory, thus not allowing subjects to remain neutral on an issue under discussion. These are all important issues that, in the fullness of time, a further development of the ideas we have presented here should address. We have been methodologically guided by the idea that we can legitimately leave open these issues, and by the hope and trust that they can ultimately be resolved in a way that is compatible with the ideas presented here.

Below we will first comment on the perspectives afforded by our four metavocabularies. Then we will turn to rational forms and how they occur in discourse, on the one hand, and in worldly states, on the other hand. Next we will rehearse our general strategies for theorizing about open reason relations in general. We will end by considering what our results mean for the understanding of logic.

6.1 A Fourfold Perspective on Reason Relations

We began this book by pointing to two traditions in the philosophy of language. One aims to understand meaning and content in terms of the norms that govern the use of language (or those that govern thought). The other aims to understand meaning and content in terms of what language (or thought) represents. We promised a reconciliation of these traditions, which are often understood as opposing each other. To explain how we have kept that promise, it is useful to think of the fourfold perspective afforded by our four kinds of rational expressive metavocabulary.

We understood the two traditions as theoretical undertakings pursued by talking about language in two different kinds of metalanguage: a pragmatic metavocabulary and a semantic metavocabulary, respectively.

They make it possible to express, to make explicit, different aspects of a single phenomenon. We can express normative relations among acts of accepting and rejecting in a pragmatic metavocabulary. And we can express the relation of worldly states making true or false what is asserted or rejected, as well as the modal relations among those states, in a semantic metavocabulary. Because the semantic metavocabulary and pragmatic metavocabulary are vocabularies for talking about the reason relations among bearers of conceptual contents, they are *meta*vocabularies. Because, and in the sense that, it is in particular the reason relations of the vocabularies for which they are metavocabularies that they address, they deserve to be called *rational* metavocabularies.

We presented an account of reason relations in a pragmatic metavocabulary in Chapters One and Two. We then showed, in Chapter Four, that this account is isomorphic to an account formulated in the semantic metavocabulary of truth-maker theory, even if we allow for failures of transitivity and monotonicity. That these two accounts share a common structure at the level of reason relations is the core of our reconciliation of the use-theoretic and the representationalist traditions in the philosophy of language.

The pragmatic-normative and the semantic-representationalist accounts codify the same reason relations, given a suitable agreement with respect to atomic sentences. Since conceptual contents are individuated by reason relations, it follows that the two accounts can capture the same conceptual contents. Both theories account for different contents by specifying exclusion relations among constellations of occurrences of these contents, and both theories posit two different ways in which contents can occur. The pragmatic-normative theory says that contents can occur in acceptances and in rejections, and that combinations of acceptances and rejections can be normatively ruled out or not. The semantic-representationalist theory says that contents can occur in worldly states that make them true and in worldly states that make them false, and that combination of truth-makers and falsity-makers can be alethically ruled out or not. We saw in Chapter Four how the explanation of the meanings of logical vocabulary that the pragmatic-normative theory gives as sequent rules and the explanation of the meanings of logical vocabulary that the semantic-representationalist theory gives as semantic clauses in truth-maker theory are isomorphic in their effects on reason relations. Indeed, it is only when one sees how the two accounts of logical vocabulary are isomorphic that the common structure of the two accounts becomes obvious.

The main differences between the theories lie in their explanations of what it means for contents to occur—in one of the two ways in which they can occur—and how and why some combinations of such occurrences are ruled out, where this ruling-out thereby exhibits different modal flavors. For instance, (the right kind of) semantic metavocabulary explains the

content expressed by non-logical base sentences by saying what features of the world those sentences represent. The sentence "The coin is made of pure copper" implies the sentence "The coin would melt at 1085 degrees Celsius" because and in the sense that it is alethically ruled out (it is impossible) that pure copper is solid at 1085 degrees Celsius, and it is incompatible with the sentence "The coin is an electrical insulator," because it is alethically ruled out for pure copper not to conduct electricity (pure copper necessarily conducts electricity). Pragmatic metavocabularies explain what is expressed by non-logical base sentences by saying what features of the discursive practices of *using* those sentences it is in virtue of which practitioners count as practically taking or treating the sentences *as* standing to one another in relations of implication and incompatibility. The sentence "The coin is made of pure copper" implies the sentence "The coin would melt at 1085 degrees Celsius" because and in the sense that it is normatively ruled out to accept the former but reject the latter (one cannot be entitled to that constellation of commitments), and it is incompatible with the sentence "The coin is an electrical insulator," because it is normatively ruled out to accept both sentences (one cannot be entitled to that constellation of commitments). The structure of these accounts is the same. What differs is the explanation of the relevant kind of ruling-out of combinations, the relevant kind of exclusion between occurrences of contents.

If we view the alethic exclusion relations between truth-makers and falsity-makers and the normative exclusion relation between acceptances and rejections as two sides of one coin, we thereby also view the pragmatic-normative and the semantic-representationalist theories of conceptual content as two sides of one coin. We can indeed view the alethic and the normative exclusion relations as two sides of one coin by holding that both pragmatic and semantic metavocabularies can be regarded as *rational* metavocabularies and, hence, as having at least one common topic, namely reason relations. Among the things they both discuss and seek to explain in their own terms are reason relations. This way to reconcile the two traditions in the philosophy of language becomes compelling if we say that what matters for having the topic of reason relations in view is the structure that is shared between the pragmatic-normative and the semantic-representationalist theories. That is metalinguistic functionalism about conceptual contents. If and insofar as this line of thought is correct, we should be interested not only in the way in which reason relations show up both in the norms that govern acceptances and rejections and in the modal relations among worldly states, but we should also be interested in the structure of reason relations in abstraction from these different ways in which they can be enmattered.

Appealing to the ways in which reason relations are enmattered gives semantic and pragmatic metavocabularies the expressive resources to provide complementary *explanations* of the reason relations of arbitrary base vocabularies. They are able to say, each in its own distinctive idiom, both what it means for some sentences to stand to others in relations of implication or incompatibility and why they do: what further facts explain them standing in just those relations. Because both of these kinds of metavocabulary appeal to conceptual resources beyond those intrinsic to the base vocabularies of which they are metavocabularies, and do so in service not just of characterizing the reason relations of those base vocabularies but of explaining them, the semantic and pragmatic metavocabularies can be called "extrinsic-explanatory" rational metavocabularies. When we consider reason relations in abstraction from the way in which they are enmattered, we move to characterizations of reason relations that do not have this explanatory dimension.

A metavocabulary for making explicit reason relations in abstraction from the different ways in which they can be enmattered is not an extrinsic-explanatory metavocabulary but an intrinsic-explicative one. This latter kind of metavocabulary for reason relations restricts itself to the conceptual resources supplied by the base vocabularies whose reason relations it characterizes. It is used to make explicit those reason relations and the conceptual contents they articulate, rather than to explain why they are as they are, or what it is for them to be as they are. The vocabulary of implication-space semantics is such an intrinsic-explicative rational metavocabulary.

The metavocabulary of implication-space semantics makes explicit reason relations in their unenmattered and pure form. In implication-space semantics, we make explicit the roles that things play in reason relations, merely in terms of those reason relations themselves. The way in which we do this is by considering the ranges of subjunctive robustness of implications, that is, the range of additions to the implication that yield or preserve a good implication. The relata of reason relations are treated as sets of implications in which only the given relatum occurs, as a premise or as a conclusion. The roles that relata of reason relations play are then defined as the equivalence classes of implications with respect to the equivalence relation of having the same range of subjunctive robustness—strictly, a pair of such roles, namely one in which the first member is the role of the implications in which only the given relatum occurs as a premise and the second is the corresponding role for the relatum occuring as a conclusion. In this way, we take reason relations and abstract from them the roles that relata play in them. We can then give an account of logical vocabulary just in terms of these roles. This account captures the common structure of the accounts of logical vocabulary given previously in semantic

and pragmatic metavocabularies, and it does so while abstracting from any particular way in which reason relations are enmattered.

Implication-space semantics provides a powerful new perspective on reason relations and conceptual contents because it allows us to move from particular bearers of roles in reason relations to those roles themselves. We can define operations on those roles, such as adjunction and symjunction, and we can define new roles in terms of old ones. These roles abstract away from any particular matter, shape, structure, or syntax that their bearers may have. For instance, two sentences that differ markedly in their syntactic structure can play the same implicational role. So can sentences in different languages, vocabularies with different sets of sentences. (In these respects, the conceptual contents of sentences as we understand them correspond to the functional classifications Wilfrid Sellars expresses with dot-quoted sentences such as ●Der Tisch ist kühl● in his original inferentialist semantics.) We can interpret the pragmatic-normative and the semantic-representationalist theories in implication-space semantics, and doing so reveals that corresponding sentences and worldly propositions in the two theories can play the same implicational roles. That is, corresponding sentences and worldly propositions play the same role in their respective reason relations. They accordingly have the same conceptual content.

Implication-space semantics reveals, in this way, the common structure of our pragmatic-normative and the semantic-representationalist metavocabularies. We take this common structure to underlie the representation relation between sentences and worldly propositions. The idea is that, for instance, the sentence "It is raining," which can occur in acceptances and rejections, represents the worldly proposition of it raining, which can occur in worldly states that make "It is raining" true and in worldly states that make it false, because the sentence and the worldly proposition have the same conceptual content—that is, they have the same implicational role.

To sum up, the metavocabulary of implication-space semantics allows us to characterize reason relations in a way that brings out the common structure of explanations of reason relations in semantic and pragmatic metavocabularies. In particular, we can now understand the pragmatic-normative theory as a theory of which discursive acts users of concepts must treat as implying each other or being incompatible with each other in order thereby to treat their sentences as representing a particular worldly proposition. Specifically, the exclusion relations among discursive acts must be isomorphic to the exclusion relations among worldly states, in spite of the different kinds of modality that articulate the reason relations of the two kinds. That is, combining acceptances or rejections of the sentence with other discursive acts and combining truth-makers or falsity-makers of the worldly proposition with other states must always be such that either both combinations are ruled out or neither is. We thus have three different

perspectives on reason relations: one in terms of the norms governing thought and talk, one in terms of the possibility or impossibility of worldly states, and one that characterizes the roles that things play in reason relations abstractly. The third perspective binds the first two together, making them visible as providing different perspectives on one topic: reason relations.

Logical vocabulary provides the fourth perspective. Indeed, it has guided our path in binding the first two perspectives together in the third one. For it is the treatment of logical vocabulary that brings out in detail how the three perspectives relate to each other. Once we see how the rules for, say, the conditional or conjunction as stated in our three metavocabularies correspond to each other, the overall correspondence between the theories formulated in these metavocabularies becomes clear. This is so because logical vocabulary makes explicit the reason relations of arbitrary base vocabularies. As a result, by finding the correspondences between the theories of logical vocabulary in our three metavocabularies, we can appreciate the correspondence between the reason relations for the entire language.

Logical vocabulary is not a metavocabulary, in the sense that it does not talk *about* reason relations or its relata. Rather, it makes reason relations explicit within (an extension of) the object language itself. The vocabulary of the sequent calculus, which we use to introduce logical vocabulary as universally LX, *is* a rational metavocabulary in the strict sense. Logical vocabulary is, nevertheless, a rational metavocabulary in the sense that it allows us to form sentences whose undeniability makes explicit what implies what and what is incompatible with what. To put it differently, there are three ways in which we can commit ourselves to an implication holding between A and B. First, we can infer B from A or, equivalently, put A forward in response to a challenge of an acceptance of B. Second, we can treat the sentence "If A, then B" as undeniable. Third, we can assert: "That A implies that B" or "'B' follows from 'A'." Logical vocabulary allows us to endorse reason relations in the second way. The other three kinds of metavocabulary allow us to endorse reason relations in the third way. Logical vocabulary and implication-space semantics have in common that they do not appeal to conceptual resources that are not provided by the reason relations that they make explicit. They both characterize reason relations in a way that is neutral both with respect to the nature of the exclusion among occurrences of contents that is essential to reason relations, and even to the nature of those occurrences themselves (the 'bearers' of conceptual contents). That is what makes them *intrinsic* rational metavocabularies. Semantic and pragmatic metavocabularies have in common that they have the resources to formulate substantive explanations of why "implies" or "follows from"

apply in particular cases. Because these explanations differ, it is not obvious that they are compatible, so that it can seem that theories formulated in semantic and pragmatic metavocabularies pull in different directions. Implication-space semantics binds together the perspectives provided by the semantic and the pragmatic metavocabularies, as it were, from above, while logical vocabulary binds them together from below.

This fourfold perspective on reason relations, conceptual content, and implicational roles is the core of the ideas that we have presented here. It takes some technical work to show how the accounts of logical vocabulary given in the three kinds of metavocabulary are all equivalent, but it can be done, and the work is richly rewarded. We are not merely waving our hands at the fourfold perspective just mentioned. We have shown that it can be treated with mathematical rigor. Such a rigorous treatment brings with it fruitful and specific clarifications of relations between our ideas and extant theories. Along the way, we have learned, for instance, how to add nonmonotonic implications to a sequent calculus while validating the deduction-detachment theorem and other desirable principles,[2] how to formulate strict-tolerant logic in truth-maker theory, how to formulate multiplicative-additive linear logic in terms of implicational roles, how to recover Correia's logic of factual equivalence from inclusion relations among implicational roles, and much more. We will come back to these fruits of the fourfold perspective that sits at the core of this book. Judging by these fruits, we think that our fourfold perspective has proven its worth.

Explicating, explaining, and proving the relations among the perspectives on reason relations and conceptual contents afforded by these four different kinds of metavocabulary also strengthens the roots of the tree that produces these fruits. As T.S. Eliot famously says in his *Four Quartets*, "We shall not cease from exploration, and the end of all our exploring will be to arrive where we started and know the place for the first time." We began our story by introducing the idea of *reason relations* of implication and incompatibility (via reasons for and reasons against) as one way of thinking about the fundamental metaconceptual bipolarity that also shows up both in the pragmatic opposition between acceptance and rejection and in the semantic opposition between truth and falsity. When they first came on the scene, reason relations were recommended as a way of thinking about language—a notion made more precise in the form of our metaconcept of a *vocabulary*, understood as a set of reason relations defined on a lexicon of declarative sentences. We have since seen that this metalinguistic rationalism, while productive, was too narrowly conceived. Language is not the only medium of reason relations. Not only what we say, but also what we talk *about* is essentially articulated by reason relations of modally robust inclusion and exclusion. Truth-maker semantic metavocabularies let us talk about nonlinguistic, modally robust, worldly matter-of-factual relations

of consequence and incompatibility. Specifically meta*linguistic* rationalism accordingly takes its place as one aspect of a broader meta*conceptual functionalism* about reason relations. That is the strategy of picking out and understanding reason relations functionally: by the complex role we have now seen them play as the demonstrably common topic of (the right sort of) pragmatic, semantic, logical, and implication-space conceptual-role model-theoretic metavocabularies.

6.2 Rational Forms

Looking at reason relations from the perspectives of our four rational metavocabularies reveals not only the essential structure of reason relations but also how roles in reason relations can be shared between different manifestations of those reason relations. We can think of the different manifestations of reason relations as different ways in which such roles are enmattered. We can accordingly think of roles in reason relations as forms that can inform different kinds of matter. That is the idea of rational forms, which is another thread that runs through the entire book.

The difference between the explanations of reason relations in the pragmatic metavocabulary of Chapters One and Three and in the semantic metavocabulary of Chapter Four is a difference in the kind of modality and a corresponding difference in the nature of occurrences of contents to which these two explanations appeal. The pragmatic explanation appeals to a normative exclusion relation between occurrences of contents that are discursive acts or attitudes. The semantic explanation appeals to an alethic exclusion relation between occurrences of contents that are worldly states. We suggest that we think of these differences as pertaining to the matter in which reason relations are manifested and that we think of what is shared between the two explanations as the forms that are enmattered in these different ways. The forms that are thus enmattered are what we call "*rational forms*."

Rational forms occur in discursive acts and also in worldly states. And since rational forms are nothing but the roles in reason relations that are propositional conceptual contents, it follows that conceptual contents occur in discursive acts and also in worldly states. In this sense, conceptual contents are real—not only in that conceptual contents occur in thought and talk but in the stronger sense that the structure of reality is a conceptual structure. That is, the structure of reality itself is articulated by reason relations. Our conceptual realism can aptly be denominated "bimodal" because the crucial difference between the two ways in which conceptual contents can occur is the difference between the kinds of modality characteristic of the exclusion relation between occurrences of contents. The idea is that the difference between kinds of matter in which

rational forms are enmattered comes with a difference between the kinds of modality that characterizes exclusion relations between occurrences of forms in the particular matter at hand. When rational forms are enmattered such that their occurrences are worldly states, then the relevant exclusion relation between occurrences is alethic. And when rational forms are enmattered such that their occurrences are discursive acts, then the relevant kind of exclusion relation is normative (deontic). This is bimodal conceptual realism. Because we understand material relations of consequence and incompatibility as reason relations, on our conception, to be a modal realist is to be a conceptual realist. That is, taking it that the world as it is objectively includes relations of necessary consequence and relative impossibility of the sort expressed in laws of nature and true subjunctive conditionals *is* taking it to have a *conceptual* structure in the sense we have explained and justified giving to that term.

Bimodal conceptual realism is *hylomorphic* in a recognizably Aristotelian sense. There are, however, important differences between Aristotelian notions of form and our rational forms. First, unlike Aristotelian forms, the metaphysical category to which bearers of rational forms belong is not primarily that of concrete particulars but that of proposition-like or sentence-like entities, namely entities whose occurrences stand to one another in reason relations of consequence and incompatibility. Second, something having a rational form does not constitute a concrete particular. Rather only *occurrences* of rational forms are concrete particulars, and what has a rational form can occur in two different ways. Third, rational forms are individuated by the modal relations between their occurrences and occurrences of other rational forms, and these modal relations can be of two different kinds, normative and alethic. So, where Aristotelians have, for the primary case of substances, forms whose manifestation in matter yields concrete particulars, we have bimodal forms whose manifestations can occur in two ways to yield concrete instances of proposition-like entities. What Aristotelian forms and our rational forms have in common is that the same form occurs in a thought and in what the thought represents. As already intimated in the Introduction to this book, this is broadly the same idea that Spinoza expresses in his slogan: "The order and connection of ideas is the same as the order and connection of things" (Spinoza, *Ethics* II, Prop. vii). The reason relations that define rational forms articulate both the order and connection of discursive acts (ideas) and the order and connection of worldly states (things).

At the end of Chapter Five we pointed out that the implication-space conceptual role semantics that is the native language of rational forms as such is both holistic and compositional. More complex conceptual roles are constructed from simpler ones by adjunction and symjunction. And the logic NMMS computes the reason relations of logically complex

sentences from those of logically simpler ones by (variants of) the usual connective rules of sequent calculi. The hoary and hackneyed opposition between coherence theories and correspondence theories of *truth* reflects a more fundamental distinction between holistic ("coherence") theories and representational ("correspondence") approaches to *meaning* or conceptual content. Rational forms are functionally defined to begin with by the reason relations linguistic or worldly bearers stand in to one another within their broadly subjective and objective realms, respectively. As with anything defined by its functional role in some larger relational whole, rational forms are accordingly identified and individuated holistically. But because rational forms can be shared between the two realms—the norm-governed discursive practices of language users and the subjunctively robust relations among worldly states—that holistic specification of conceptual contents builds in the dimension of correspondence that we are calling "bimodal conceptual realism." Our view is accordingly distinctive in locating the most basic correspondence between words and the world in reason relations, and thereby in rational forms or propositional conceptual contents. This fundamental representational dimension only becomes visible in terms of the relations between what is expressed in pragmatic metavocabularies specifying subjects' use of expressions in deontic normative terms and semantic metavocabularies specifying the objective world represented in alethic modal terms. It is not reducible to, or intelligible just in the terms of, the notion of representation codified in the interpretation functions that truth-maker semantic metavocabularies specify. The meta-metalinguistic character of our account of representation as the amphibiousness of rational forms between language and the world, with its essential appeal to pragmatic metavocabularies for specifying the use of linguistic expressions, operates in a metaconceptual space that, in spite of its historical credentials, is essentially richer than talk about representation or correspondence in terms of truth or meaning.

Rational forms provide a way to understand why and how what is expressed by declarative sentences is both what can be semantically evaluated as true or false and what can be pragmatically accepted or rejected. For what is expressed by declarative sentences are propositional conceptual contents or, equivalently, rational forms. If a rational form occurs positively in a worldly state, then this makes what the sentence expresses true. And if it occurs negatively, this makes what the sentence expresses false. If a rational form occurs positively in a discursive act, then this act is an acceptance of what the sentence expresses. And if the rational form occurs negatively in a discursive act, then this act is a rejection of what the sentence expresses. Thus, what is expressed by declarative sentences is what is made true or false by worldly states, and it is also what is accepted and rejected in discursive acts.

Rational forms have a dual bilaterality. They have two sides in that they can occur in two opposing ways, positively and negatively. And both of these sides can occur in two different kinds of matter, the matter of worldly states related by alethic exclusions and the matter of discursive acts related by normative exclusions. The duality of having a positive and a negative way to occur reflects the fact that rational forms are articulated by relations of reasons for and reasons against. The duality of occurring in two kinds of matter reflects the isomorphism of reason relations in pragmatic and in semantic metavocabularies. In semantic terms, we can say that the first duality corresponds to the contrast between truth and falsity, while the second duality corresponds to the contrast between representings and what is represented. The space in which rational forms can occur is the field spanned by these four corners.

The development of the notion of rational forms was the main result of Chapter Four. There we showed how we can understand the normative bilateralism of Restall and Ripley, on the one hand, and Fine's bilateralism of truth-makers and falsity-makers, on the other, as two sides of reason relations and, hence, of the rational forms they articulate. Developing this notion yielded as a by-product a truth-maker semantics for strict-tolerant logic, as well as a generalization of both strict-tolerant logic and truth-maker theory to open, nonmonotonic and nontransitive consequence relations. Along the way, we saw how proof-theoretic sequent rules correspond in a fine-grained way to the semantic clauses of truth-maker theory (and, later, to constructions in implication-space semantics). And we formulated a novel way to understand consequence in truth-maker theory in modal terms, thus forging a connection to modalism about consequence and the long tradition behind it. Developing the big philosophical idea of rational forms that are shared between representations and what is represented thus paid significant dividends at the level of technical details. The simultaneous work on the big philosophical issue and the technical details gives us a notion of rational form that has the potential to illuminate the subject-object nexus from a perspective that is not only new, but is one that we can also manipulate with mathematical precision and rigor.

6.3 Theorizing Open Reason Relations

One of our overarching aims in this book was to develop methods for theorizing open reason relations, that is, reason relations in which monotonicity and transitivity can fail. More specifically, our aim was to develop such methods in a way that is consistent with semantic inferentialism and logical expressivism. This is a thread that binds together the three ideas with which we opened this chapter.

Implication-space semantics, which we have presented in Chapter Five, is the most abstract way to theorize open reason relations. Implication-space semantics abstracts away from the duality of two kinds of matter in which rational forms can occur and characterizes rational forms independently of the matter in which they can occur. This requires abstracting away from the substantive concepts *extrinsic* to the base vocabularies for which they are metavocabularies that permit semantic and pragmatic *explanations* of reason relations. The implication-space metavocabulary provides a model-theoretic semantics for the conceptual roles bearers of conceptual contents play in virtue of standing to one another in reason relations of implication and incompatibility. It is a reason-based inferentialist semantics, rather than a truth-based representational semantics such as truth-maker semantics.

If the devil is in the details of different kinds of matter, then the angels will be found working with rational forms at the more abstract (intrinsic) level. While we characterize reason relations in the pragmatic and semantic metavocabularies in terms of a particular modal exclusion relation, implication-space semantics includes a materially undifferentiated and primitive distinction between good and bad implications. It is materially undifferentiated in the sense of being amphibious between the characterizations in a normative pragmatic and in an alethic modal semantic metavocabulary. While we characterize occurrences of contents as acceptances and rejections and as truth-makers and falsity-makers in the pragmatic and semantic metavocabularies, respectively, implication-space semantics includes a materially undifferentiated and primitive structural bipolarity of positive (premisory) and negative (conclusory) occurrences of conceptual contents. And while we characterize combinations of occurrences of contents as discursive positions or as fusions of states in the pragmatic and semantic metavocabularies, respectively, implication-space semantics includes an undifferentiated and primitive mode of combining implications. As the pure (intrinsic) metavocabulary of conceptual roles, inferentialist implication-space semantics is our candidate for the language of the angels, who immediately grasp the abstract rational forms shared by worldly states on the objective side of the intentional nexus and discursive acts on the subjective side. For us fallen creatures, the intrinsic-explicative metavocabularies of implication-space semantics and logic are tools for coming closer to angelic rational self-consciousness. For they allow us to become reflective and critical—in our limited way—with respect to reason relations, independently of the way in which they are enmattered.

The three kinds of abstraction just sketched allow implication-space semantics to capture the essential structure of reason relations. And we saw in Chapter Five that this is the structure of a commutative monoid defined on a set of pairs, together with a partition of that set of pairs.[3] The primitive distinction between good and bad implications shows up as the partition

of the monoid set by the particular subset. That conceptual contents can occur in two different ways shows up in the fact that the elements of the monoid are pairs. And the primitive way to combine implications shows up as the monoidal operation. These are the basic ingredients one needs to build an implication-space model in which one can interpret bearers of conceptual contents, such as sentences.

We can think of the construction of such a model as happening in two stages. At the first stage, what is interpreted, as well as the building blocks of the interpretants, are implications. Thus, implication-space semantics is an inferentialist semantics not only in that its semantic interpretants are implicational roles but also in that what is interpreted are, in the first instance, candidate implications. The semantic interpretation of a candidate implication is its equivalence class with respect to its range of subjunctive robustness, that is, the class of implications that have the same range of subjunctive robustness. Our monoidal structure provides the resources to define ranges of subjunctive robustness because we can say that the range of subjunctive robustness of an element of our monoid set is the set of elements such that when combined by the monoidal operation with the target element yield a result in the distinguished subset. Therefore, as long as candidate implications form a commutative monoid of pairs with a particular subset of good implications, we can define ranges of subjunctive robustness and, hence, define the semantic interpretants of implications.

The ranges of subjunctive robustness of candidate implications are their "goodness" conditions, as truth conditions are the "goodness" conditions of sentences in truth-conditional semantics. For a candidate implication to be good in the reasons-first semantic setting is for its premises to provide reasons for its conclusion, while for a sentence to be good in the truth-first semantic setting is for it to be true. The advance from a conception of semantic goodness to a conception of meaning is the advance to consideration of circumstances under which a reason relation or sentence *would* be good. In truth-maker theory, the circumstances are sets of worldly states. In implication-space semantics, the circumstances are additional premises and additional conclusions that would make or keep the implication good. The semantic interpretation of the implication is then the class of implications that have the same conditions of semantic goodness.

At the second stage, we interpret particular bearers of conceptual contents, such as sentences. The semantic interpretant of a sentence (or other particular bearer of a conceptual content) is its implicational role, and the implicational role of a sentence is a pair whose first element is the sentence's premisory role and whose second element is the sentence's conclusory role. The premisory role is the semantic interpretant of the candidate implication that is a bare occurrence of the sentence as a premise,

and the conclusory role is the semantic interpretant of the candidate implication that is a bare occurrence of the sentence as a conclusion.

In this way, implication-space semantics isolates the roles that its relata play in any given reason relation. It is a way of arriving at the implicational roles of things that stand in reason relations by means of mathematical abstraction. Crucially, this works for structurally open reason relations as well as for topologically closed ones. Indeed, it works not only for reason relations in which Monotonicity and Transitivity can fail but also for reason relations in which Contraction and Reflexivity can fail. It is accordingly an extremely versatile and flexible tool for codifying the implicational roles that the relata in open reason relations play.

These aspects of implication-space semantics apply to open reason relations even before we consider logical vocabulary. The reason relations implication space-semantics codifies need to be closed neither in the sense of being a topological closure operator nor in the sense that explicitation of implications is inconsequential. That is, these reason relations need to obey neither Monotonicity, Cut, and Reflexivity, nor the weaker principles of Cautious Monotonicity and Cumulative Transitivity. So if Γ implies A, then adding A to Γ can change what is implied in both directions: by losing consequences and by adding consequences. That is, if—in addition to A—the premises Γ also imply B but do not imply C, then the combination of Γ and A can nevertheless fail to imply B, and it can imply C. So making the implications of a set of premises explicit in the sense of adding them to the premise-set can have substantive effects on what is implied. In this sense, the kind of inference that consists in acknowledging commitment to implications of one's commitments by taking them as further explicit premises is a creative act. It may open up paths to implications one has not foreseen, and close alternative ways to develop one's commitments. Equally reasonable paths of explicitly endorsing implications of the same starting point can lead to incompatible end points. As already intimated in Chapter Two, this rational hysteresis contains the seed of the fact that it is essential to rational beings to have not merely a past but a history. There need not be any single end-point on which all fully rational creatures will converge when they draw out the consequences of a common starting point, and whether the position at which some rational creatures arrived in this way is reasonable depends in part on the path that led them there. Rationality can be a matter of how one got to where one is, not merely in the psychological sense of depending on the particular reasons for which one undertook a given commitment, but in the sense that different reasons are available on different paths from a starting point to an end point.

Developing the tools for theorizing open reason relations in a formally rigorous way has proven to be fruitful in unexpected ways. Only by doing so could we use Girard's phase-space semantics for linear logic as a stepping

stone to the essential structure of reason relations. And it is only once one sees that this structure is the structure of a monoid defined on a set of pairs with a partition on it that one can clearly see this structure in the theories formulated in pragmatic and semantic metavocabularies. As a by-product of this insight, it becomes clear that Girard's phase-space semantics uses the notion of the dual of facts to do two tasks that we ought to keep separate, namely the task of giving an account of negation and the task of defining the ranges of subjunctive robustness that define implicational roles. And we can prevent conflating these two tasks by using pairs as the elements of our monoid set, and letting the structure of pairs underlie our account of negation. It was this formal investigation that cleared the path to a deeper philosophical understanding of open reason relations.

6.4 The Logics of Open Reason Relations

One of many uses that one can make of implication-space semantics is to compute the implicational roles of arbitrary logically complex sentences from the implicational roles of logically atomic sentences of any base vocabulary—even when the reason relations of the base vocabulary, and so its (conservative) logical extension, are radically substructural. We do this by defining the operations of adjunction and symjunction on implicational roles. Adjunction combines positive or negative occurrences of conceptual contents in a single implication and returns the implicational role of that combined implication. It is a way to look at the joint effects of several occurrences of different conceptual contents in a single implication. Symjunction looks at the intersection of the ranges of subjunctive robustness of roles and returns the implicational role whose members have that intersection as their range of subjunctive robustness. It is, as it were, a way to look at the overlap of, or at what is shared between, the implicational roles of several implications. Thus, adjunction combines occurrences of conceptual contents in single implications and takes the implicational role of the result, whereas symjunction combines the roles of several implications by looking at the intersection of their ranges of subjunctive robustness. With these operations on the table, we can say, for instance, that the conclusory implicational role of a conditional is the adjunction of the premisory implicational role of the antecedent and the conclusory implicational role of the consequent. And we can say that the implicational role of a negation is the result of swapping the premisory and the conclusory implicational roles of the negatum.

There is a close connection between the way in which the implicational roles of logically complex sentences are computed in model-theoretic implication-space semantics and the proof-theoretic sequent rules for the logical connectives. The specification of the premisory role of a logically

complex sentence corresponds to the right-rule for the principal connective of the sentence, and the specification of the conclusory role corresponds to the left-rule. Adjunction corresponds to sequent rules with just one top sequent, and symjunction corresponds to rules with several top sequents. If an adjunction or symjunction operates on a premisory role, then the sentence with that role occurs on the left side in the (or a) top sequent. And if an adjunction or symjunction operates on a conclusory role, then the sentence with that role occurs on the right side in the (or a) top sequent. In this way, one can read off sequent rules from clauses for implicational roles, and one can read off clauses for implicational roles from sequent rules. Indeed, our brief discussion of multiplicative-additive linear logic in Chapter Five showed that this mapping can be extended from the Ketonen sequent rules to sequent rules for the multiplicative and additive connectives of linear logic.

There is also a close connection between the way in which the implicational roles of logically complex sentences are computed in implication-space semantics and the semantic clauses of truth-maker theory. The specification of the premisory role of a logically complex sentence corresponds to clauses for verifiers in truth-maker theory, and the specification of the conclusory role corresponds to the clauses for falsifiers. Adjunction corresponds to truth-maker clauses in which the states that verify or falsify the compound sentence are fusions of two states, and symjunction corresponds to truth-maker clauses that specify the states that verify or falsify the compound sentence by a disjunction. Appeals to verifiers on the right-hand side of truth-maker clauses correspond to appeals to premisory roles, and appeals to falsifiers on the right-hand side of truth-maker clauses correspond to appeals to conclusory roles.

There is an equally simple correspondence for the structural rules. The Weakening rule in the sequent calculus corresponds to Downward-Closure in truth-maker theory and to the requirement that all ranges of subjunctive robustness be maximal in implication-space semantics. The principle of Containment in the sequent calculus corresponds to Exclusivity in truth-maker theory and it corresponds to the principle that pairs with overlapping elements are good implications in implication-space semantics. The Cut rule in the sequent calculus corresponds to Exhaustivity in truth-maker theory, and in implication-space semantics it corresponds to the principle that if an implication's range of subjunctive robustness includes the symjunction of the premisory and the conclusory role of some one thing, then the implication is good.

By appreciating these correspondences, the underlying shared structure of reason relations becomes visible in all of the theories. Implication-space semantics captures this structure in the clearest and most abstract way, by allowing us to talk directly about roles in reason relations, rather

than merely the relata in reason relations that play these roles. However, once one sees the common structure, it is easy to identify these roles in each of the different theories. As already intimated, the roles of logically complex sentences guide the way here because they make explicit arbitrary material reason relations. Once we see the correspondences in the case of logically complex sentences, the general isomorphism between the structures becomes difficult to overlook.

In a final additional step of abstraction, we can define inclusion relations between implicational roles in terms of set-theoretic inclusion relations between their ranges of subjunctive robustness. The idea is that, relative to some condition on implication-space models, an implicational role is included in another implicational role if and only if, in all implication-space models that meet the condition, the range of subjunctive robustness of the former role is a subset of the range of subjunctive robustness of the latter role. If this holds, one can (in all these models) replace bearers of the former role by bearers of the latter role *salva consequentia*, that is, without turning a good implication into a bad one. Thus, implicational role inclusion relations codify relations of substitutability of bearers of these roles in implications *salva consequentia*.

The vocabulary of such role inclusion relations is an intrinsic rational metametavocabulary, built on top of the implication-space model-theoretic semantics for conceptual roles. It provides the expressive power to make explicit metainferential reason relations among implicational roles. In particular, we can recover the notion of local metainferential validity in terms of implicational role inclusions. This offers an illuminating new semantic perspective on the relations among a variety of well-studied logics. The paraconsistent logic of paradox (LP) shows up as the logic of conclusory role inclusions. That is, A implies B in LP if and only if the conclusory role of A is included in the conclusory role of B, in all conic implication-space models. And the dual paracomplete strong Kleene logic (K3) shows up as the logic of premisory implicational role inclusions. That is, A implies B in K3 if and only if the premisory role of A includes the premisory role of B, in all conic implication-space models. And we can formulate strict-tolerant and tolerant-strict logic in a similar way. Moreover, Correia's logic of factual equivalence results from our formulation of K3 by dropping the constraint that models be conic but require that the implication goes in both directions. And Correia's dual logic results from our formulation of LP in the analogous way. Indeed, we can even recover analogues of the notions of essence and grounding in terms of implicational role inclusions.

The laws of all these familiar logics show up in implication-space semantics as licenses to substitute bearers of certain roles *salva consequentia*. For that is what inferential role inclusions are. This is

how what is currently studied under the heading of metainferential validity emerges in implication-space semantics. However, implication-space semantics is much more flexible and powerful than the usual three-valued semantic frameworks in which many of the above-mentioned logics are usually studied. In contrast to the usual three-valued setting, we can codify not only nontransitive but also nonmonotonic consequence relations in implication-space semantics. Similarly, we can recover Correia's logics, which cannot be formulated in a standard three-valued setting. Similarly, the connection to truth-maker theory and to linear logic cannot be appreciated from the perspective of a standard three-valued setting. By contrast, if one adopts our fourfold perspective on reason relations, all these rich and interesting connections become visible. Thus, we think that we have not only found a way to theorize open reason relations but a genuinely eye-opening way to do so.

Looking back, we can see the structure revealed by implication-space semantics in the NMMS sequent calculus from Chapter Three—a structure that cannot easily be seen by looking merely at that calculus. However, the sequent calculus brings out very clearly the idea of treating the material implications of nonlogical vocabulary as the given basis to which we then add logical vocabulary. For in the sequent calculus, we treat the material implications (and incompatibilities) as axioms, which are then closed under the sequent rules. In this way, the complete logically extended vocabulary (lexicon of logically complex sentences plus reason relations among them) can then be computed from the base vocabulary. That we can close arbitrary sets of nonlogical axioms under these sequent rules is what it means that the logically extended vocabulary can be *elaborated from* any arbitrary base vocabulary.

One thing one can reasonably mean by the "logic" of this logical extended vocabulary is the implications and incompatibilities that hold in every logical extension of any base vocabulary that obeys Containment. These are sequents such as $\Gamma, A \rightarrow B, A \mathrel{\vdash\mkern-7mu\sim} B, \Delta$ that hold no matter what implications hold in the base vocabulary, simply in virtue of the rules for determining the reason relations that logically complex sentences stand in, together with Containment. This logic is just classical logic. So, in this sense, the logic of open reason relations that we are suggesting is classical propositional logic.

Classical logic is special because the constraint of Containment can plausibly be taken to be an essential part of the structure of conceptual contents. For, according to the view that we are suggesting, conceptual contents have essentially two opposing sides, and Containment can be understood as saying that the two sides of any genuine conceptual content always exclude each other, no matter in what context they occur. Thus, Containment can plausibly be understood as reflecting the opposition

between the two sides of contents. If one accepts this idea, then classical logic is a formulation of the reason relations that hold in all logically extended vocabularies. This is what we think the tradition got right in putting classical logic at the center of logic and the study of reason relations. It would be a mistake, however, to ignore the richer open reason relations within which classical logic emerges as a minimal part.

The sequent calculus of Chapter Three also suggests several different clear ways to understand the claim that logical vocabulary makes explicit arbitrary reason relations. One way to understand this central claim of logical expressivism is this: A sentence ϕ makes explicit that Θ is a reason for Λ if and only if, for all Γ and Δ, we have $\Gamma \mathrel{|\!\!\sim} \phi, \Delta$ just in case $\Gamma, \Theta \mathrel{|\!\!\sim} \Lambda, \Delta$. And the sentence ϕ makes explicit that Θ is a reason against Λ if and only if, for all Γ and Δ, we have $\Gamma \mathrel{|\!\!\sim} \phi, \Delta$ just in case $\Gamma, \Theta, \Lambda \mathrel{|\!\!\sim} \Delta$. We saw in Chapter Three that the following conditions jointly ensure that logical vocabulary allows us to form sentences that can make arbitrary reason relations explicit in that sense.

Deduction-Detachment (DD) Condition on Conditionals:
$\Gamma \mathrel{|\!\!\sim} A \to B$ if and only if $\Gamma, A \mathrel{|\!\!\sim} B$.

Incoherence-Incompatibility (II) Condition on Negation:
$\Gamma \mathrel{|\!\!\sim} \neg A$ if and only if $\Gamma, A \mathrel{|\!\!\sim}$ if and only if $\Gamma \# A$.

Antecedent-Adjunction (AA) Condition on Conjunctions:
$\Gamma, A, B \mathrel{|\!\!\sim} \Delta$ if and only if $\Gamma, A \wedge B \mathrel{|\!\!\sim} \Delta$.

Succedent-Summation (SS) Condition on Disjunctions:
$\Gamma \mathrel{|\!\!\sim} A, B, \Delta$ if and only if $\Gamma \mathrel{|\!\!\sim} A \vee B, \Delta$.

That these conditions hold can be appreciated from the perspective of truth-maker theory and implication-space semantics. In the sequent calculus setting, however, that these conditions hold follows immediately and transparently from the sequent rules of NMMS (and the variants we have presented), and the fact that they are invertible. Insisting that these conditions hold, even in logically extended open reason relations, yields results that are familiar in many ways but unorthodox in other ways. Some unorthodox consequences are, for instance, that premises can imply a conditional and its antecedent without implying its consequent (that is, *meta-modus-ponens* can fail), and that sentences that are equivalent in classical logic cannot always be substituted for each other *salva consequentia*. We hope to have shown over the course of the book that accepting such unorthodox consequences not only gives us logical vocabulary that can make arbitrary reason relations explicit, but also opens the door to new and illuminating perspectives on representation, the

relations between semantic and pragmatic metavocabularies, the essential structure of reason relations, and a rich network of surprising connections among familiar logical theories.

In Chapter Three, we also offered another sense in which logical vocabulary makes explicit the reason relations of arbitrary base vocabularies. The idea is that every reason relation among logically complex sentences reflects—and thereby expresses—specific reason relations among sentences of the base vocabulary. We saw that the version of NMMS in which Contraction holds fulfills this task in one direction: For any sequent of the NMMS-extension of any base vocabulary, the sequent is derivable if and only if a unique and easily specifiable set of sequents is in the base consequence relation. In order to fulfill this task also in the other direction, we must move to the version of NMMS in which Contraction fails. In that version, for every arbitrary set of sequents, X, that can be formulated in the language of the base vocabulary, there is a single sequent in the logically extended vocabulary such that this sequent is derivable if and only if all the sequents in X are in the base consequence relation. So, in the noncontractive version of NMMS every collection of base sequents can be represented by a single sequent in the logical extension. This result also applies, of course, to the implication-space formulation of the noncontractive version of NMMS. Hence, the theories that we have put forward in this book treat logical vocabulary in such a way that it also makes arbitrary reason relations explicit in the second of the two senses (though this holds only in one direction for the contractive versions).

Putting the two points together, we can say: NMMS allows us to elaborate logical vocabulary from arbitrary base vocabularies, and the logical vocabulary of NMMS makes arbitrary reason relations of any given base vocabulary explicit. We take the combination of these two facts to mean that the theories that we have presented in this book live up to the ideal of logical expressivism. They underwrite the claims that logical vocabulary is *el*aborated from and *ex*plicative of (LX for) the reason relations of arbitrary base vocabularies. Because it can extend any and every base vocabulary (that meets the most minimal conditions), the ideal logic is *universally* LX. The theories we have presented approach this ideal of being universally LX.

As a by-product of developing these ideas in a rigorous way, we learned how to introduce operators that make explicit when implications hold monotonically (in a generally nonmonotonic setting). And we also showed how the strategy can be generalized to make explicit both the classicality of an implication (in a generally nonclassical setting) and when contraction holds (in a generally noncontractive setting).

To sum up, there is a sense in which the logic of open reason relations is simply classical logic. However, our fourfold perspective on reason

relations revealed many other familiar logics in the structure of reason relations. In particular, since Cut can fail, we can think of the classical consequence relation that emerges as the logic of open reason relations as really being a formulation of strict-tolerant logic. Moreover, the logic of paradox, for instance, is the logic of conclusory role inclusions, in conic models, and Correia's dual logic results if we look at all models and not just the conic ones. And strong Kleene logic stands in the analogous relation to Correia's logic for premisory role inclusions. Furthermore, if we move from Containment to Reflexivity and allow for failures of Contraction, then the structure of implication-space semantics, in effect, coincides with the structure of phase-space semantics for multiplicative additive linear logic, and we can define the linear connectives in that setting. All of these connections hold despite the fact that we are far less exclusive in the reason relations that we consider than virtually all of these other logics. The logic of paradox and strong Kleene logic are monotonic, Correia's logics and linear logic are transitive, and none of these theories considers material implications and incompatibilities. None of these logics is *the* logic of open reason relations. Rather, they all capture specific aspects of open reason relations. Seeing open reason relations from the perspective of the four rational metavocabularies in the way we have suggested provides a clear and detailed picture of the different aspects that these logics capture.

6.5 Final Remarks

This is a book about reasons and, in particular, reason relations. It is animated by a kind of rationalism about philosophy, language, logic, and meaning that consists in the interlocking convictions that:

• Concern with the nature of reasons is coeval with philosophy, and abides at its core.
• Language properly comes to be a central focus of philosophical attention insofar as it is understood as a medium of reasons.
• The methodological promise of logic as a tool for philosophical understanding stems from the idea that it expresses the essential form of reasons as such.
• Meaning matters in philosophy because the specifically conceptual contents of declarative sentences consist in the role they play in reasoning.

Adopting a pragmatics-first order of explanation, we pursued this reason-centered approach by distinguishing discursive practices as those in which some performances have the significance of assertions and denials, expressions of commitment to accept or reject claimables. That

significance we understand as depending on practices of defending and challenging claimings, giving reasons for and against them, thereby affecting practitioners' entitlements to their commitments. This pragmatic discursive rationalist account gave us our first glimpse of and grip on doxastic reason relations of implication and incompatibility among claimables. Along the way from there we have seen how to implement and fill in the details of our further strategic commitments, to understand reason relations and conceptual contents in accordance with the principles of logical expressivism, semantic inferentialism, and bimodal conceptual realism.

The result is an intricately structured quartet of rational metaconceptual vocabularies, that is, vocabularies for specifying reason relations and conceptual contents: a bilateral normative pragmatic vocabulary, two kinds of model-theoretic semantic vocabulary (one representationalist and one inferentialist), and a particularly expressively powerful but otherwise familiar logical vocabulary presented in a sequent-calculus format. It is worth rehearsing some of the relations among the members of this quartet of vocabularies for explicitly expressing reason relations and conceptual contents. To begin with, we offered reasons to think that they represent something like a complete set of kinds of rational metavocabularies. The bilateral pragmatic metavocabulary and the truth-maker semantic metavocabulary are extrinsic to and explanatory of the reason relations of the base vocabularies they address, while the logical and implication-space vocabularies are intrinsic to and explicative of the reason relations of the base vocabularies from which they are elaborated. Within those two broad kinds, the pragmatic and semantic metavocabularies address the practices of using declarative sentences of ground-level vocabularies and their representational meanings, respectively. Logical vocabulary expresses the reason relations of arbitrary base vocabularies by extending those very vocabularies, while the implication-space metavocabulary codifies those reason relations and the conceptual contents they articulate in a new metavocabulary built out of raw materials supplied by the base, but mentioning rather than using the expressions of that base vocabulary.

We have shown how metaconceptual vocabularies of all these four general kinds can be so constructed that they have the expressive power to make explicit the reason relations (and the rational forms or conceptual contents they articulate) not only of structurally closed base vocabularies—the kind the philosophical tradition by and large restricts itself to considering—but also of those that have structurally open or substructural nonmonotonic (and even hypernonmonotonic) and nontransitive reason relations. At the center of this achievement is the logic NMMS. There are many formulations of classical logic in the proof-theoretic vocabulary of the sequent calculus that are all equivalent in the

context of the full topological structure that Gentzen and Tarski both discern in and impose on logical consequence relations. In substructural settings, these specifications come apart and behave differently. We exploit that diversity to find the expressive sweet spot occupied by NMMS. Ketonen's invertible connective rules, with judicious mixing of additive and multiplicative rules (usually avoided in order to secure the admissibility of the structural transitivity principle of Cut), remain well-behaved and exhibit hitherto unprecedented expressive power when we use them to extend not only structurally closed but also more relaxed open-structured base vocabularies. The purely logical reason relations of the logically extended vocabularies that result from base vocabularies with nonmonotonic and nontransitive reason relations are wholly classical—and, indeed, structurally closed: monotonic and transitive. In this sense we offer not a nonmonotonic (or, more generally, substructural) logic, but a logic for expressing nonmonotonic (and, more generally, substructural) material implications and incompatibilities.

Following the clues provided by this bilateral pragmatics for base vocabularies with open-structured reason relations, we showed how to define consequence in the truth-maker semantic metavocabulary so that it is sound and complete for our logic—and thereby how to extend the truth-maker framework to codify open-structured, nonmonotonic and nontransitive material reason relations. The implication-space metavocabulary also supplies a sound and complete semantics for NMMS, and for open-structured reason relations generally. Following that clue, we showed how to construct a thorough-going isomorphism between the extrinsic-explanatory representational truth-maker semantic metavocabulary and the intrinsic-explicative implication-space semantic metavocabulary. Any constellation of reason relations, open or closed, that can be expressed by the one kind of semantic metavocabulary can be expressed by the other. This is a deep connection between representationalist model-theoretic semantic frameworks and inferentialist model-theoretic semantic frameworks. In addition, the interpretation we offered of multiplicative and additive linear logic in implication-space semantics made visible a fundamental homology between the basic operations on conceptual roles in that semantics (adjunction and symjunction, premisory and conclusory roles) and basic features of sequent-calculus proofs (such as single-premise-sequent and multi-premise-sequent connective rules, left and right rules)—a structural connection between the proof theoretic sequent-calculus metavocabulary in which we construct our logic and the implication-space model-theoretic metavocabulary that we take to be the intrinsic semantic metavocabulary of reason relations.

The right kind of logical vocabulary, NMMS, accordingly ties together the other three kinds of rational metavocabulary. The bilateral normative pragmatic metavocabulary on which our more deontically fine-grained version is based was introduced in the first place to explain the implication relations expressed as sequents in multisuccedent proof-theoretic sequent calculi for various logics. Both the truth-maker semantics and the implication-space semantics are sound and complete for that logic, and seeing why and how they are reveals an isomorphism between them at the level of reason relations. The final tie that binds together the quartet of rational metavocabularies is the isomorphism between the bilateral pragmatic metavocabulary and the truth-maker semantic metavocabulary when just the right conception of consequence is defined in the latter framework. A premise-set Γ implies a conclusion-set Δ in the deontic pragmatic sense if and only if commitment to accept all of Γ precludes entitlement to deny all of Δ, and in the alethic semantic sense if and only if every fusion of truth-makers of all of Γ with false-makers of all of Δ is an impossible state. Working out this detailed isomorphism between reason relations specified in terms of norms governing subjects' use of sentences and what, in virtue of that very isomorphism, we can see as the same reason relations specified in terms of the worldly representational meaning of those sentences (what they say about how things objectively are) is offering an account of the relations between language and the world. In our account, the representational language-world nexus is to be understood at the deepest level in terms of reason relations, and only later, and as a consequence, at the level of sentences and what makes them true or false. That account makes visible the sense in which rational forms, articulated by relations of consequence and incompatibility, can be shared by what people say and the world they are talking about. The implication-space semantic metavocabulary makes those rational forms explicit. It is their native language.

In closing let us offer one final perspective on this constellation of rational metavocabularies. The discursive rationalism that motivates and shapes our enterprise understands vocabularies in general as the medium of consciousness. "Consciousness" in this sense is conceptual awareness, Kantian apperception: sapience rather than the mere sentience that consists in the capacity to have raw feels such as pains and sensations of red. Being conscious in this sense is being able to accept or reject, assert or deny, say, think, believe or suppose that p, where p is some declarative sentence. Consciousness so understood is a matter of responding to things by applying concepts, making claims. The claimable, conceptual contents expressed by using declarative sentences to assert and deny, and to challenge and defend assertions and denials, are what they are because of the reason relations of implication and incompatibility they

stand in to other such conceptual contents. Every set of such reason relations is accordingly a form of rational consciousness. The four kinds of vocabulary we have considered for making explicit in different ways the reason relations of arbitrary base vocabularies are accordingly forms of rational *self*-consciousness: rational consciousness of what rational consciousness is. Each of these rational metavocabularies adopts and expresses a distinctive perspective on reason relations: pragmatic, logical, representational-semantic, and inferential-semantic. The strategy of metaconceptual functionalism is to understand reason relations as whatever one can understand in these four intricately interrelated ways. By considering the relations among all of them, the final functional account of reason relations is a higher, more complete form of rational self-consciousness than any of the four kinds of rational metavocabulary provides on its own. Metaconceptual rationalism as filled in here accordingly shows up as an account of the essential internal structure of rational self-consciousness as such.

Notes

1 Our motivation for this is, of course, related to a variant of Frege's context principle. Hence, we don't think of this as a culpable omission but rather as laying the foundations for an investigation of subsentential expressions. A defense of this claim is, however, beyond the scope of this book.

2 As a reminder of just one of the many rare virtues that our theories have, recall, for instance, that our disjunction obeys so-called "disjunction simplification," which says that if $\Gamma, A \vee B \mathrel{\vdash\!\!\!\sim} \Delta$, then $\Gamma, A \mathrel{\vdash\!\!\!\sim} \Delta$ and $\Gamma, B \mathrel{\vdash\!\!\!\sim} \Delta$. And as Fine (1975) pointed out, no theory of nonmonotonic consequence that allows intersubstitutions of sentences that are classically equivalent can do that.

3 Here we followed Daniel Kaplan, who showed in his PhD dissertation (Kaplan, 2022), how one can generalize Girard's phase-semantics for linear logic so as to allow for material implications and failures of Cut.